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Extraction of In-Medium Nucleon-Nucleon Amplitude From Experiment

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Abstract

The in-medium nucleon-nucleon amplitudes are extracted from the available proton-nucleus total reaction cross sections data. The retrieval of the information from the experiment makes the estimate of reaction cross sections very reliable. Simple expressions are given for the in-medium nucleon-nucleon amplitudes for any system of colliding nuclei as a function of energy. Excellent agreement with experimental observations is demonstrated in the ion-nucleus interactions.

1. Introduction

The presence of other nucleons in the nucleus significantly modifies free two-body scattering. This is mainly effected by the Pauli suppression in the intermediate stages, which brings in the density dependence (ref. 1) of the interaction in a very natural way. The detailed microscopic reaction matrix calculations (refs. 1–3) are time consuming. It is, therefore, desirable to extract, directly from experiment, the renormalization of free nucleon-nucleon cross sections in the medium. This method, although not completely microscopic in nature (in that it extracts renormalization from the experiments), does include several renormalization effects, which are difficult to calculate in a microscopic theory. In addition, it has the advantage that the modifications are directly linked to the experimental observations, and hence can be used with ease (and with more confidence) in explaining experimental data.

2. Method

The coupled-channel approach developed at Langley Research Center is followed. In this method, the Schrodinger equation for heavy ion scattering can be solved in the eikonal approximation (refs. 4–9), resulting in the following matrix of scattering for elastic amplitudes:

$$f(\mathbf{q}) = -\frac{ik}{2\pi} \int \exp(-i\mathbf{q} \cdot \mathbf{b}) \{ \exp[i\chi(\mathbf{b})] - 1 \} d^2b \quad (1)$$

where f and χ represent matrices, k is the projectile momentum relative to the center of mass, \mathbf{b} is the projectile impact parameter vector, \mathbf{q} is the momentum transfer, and $\chi(\mathbf{b})$ is the eikonal phase matrix (see ref. 7 for detailed expressions), which is calculated using the two-body amplitude f_{NN} parameterized as

$$f_{NN}(\mathbf{q}) = \frac{\sigma(\alpha + i)}{4\pi} k_{NN} \exp\left(-\frac{Bq^2}{2}\right) \quad (2)$$

Here k_{NN} is, again, the relative wave number in the two-body center of mass system, σ is the two-body cross section, B is the slope parameter, and α is the ratio of the real part to imaginary part of the forward, two-body amplitude.

The total cross section is found from the elastic amplitude by using the optical theorem as follows:

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \text{Im} f(\mathbf{q} = 0) \quad (3)$$

Equations (1) and (3) give

$$\sigma_{\text{tot}} = 4\pi \int_0^\infty b db \{ 1 - \exp[-\text{Im}(\chi)] \cos [\text{Re}(\chi)] \} \quad (4)$$

The total absorption cross section σ_{abs} is given by

$$\sigma_{\text{tot}} = \sigma_{\text{abs}} + \sigma_{\text{el}} \quad (5)$$

where σ_{el} is the total elastic cross section. Integrating equation (1) by using $d\Omega \approx d^2q/k^2$ and by using equations (4) and (5) yields (ref. 7)

$$\sigma_{\text{abs}} = 2\pi \int_0^\infty b db \{ 1 - \exp[-2\text{Im}(\chi)] \} \quad (6)$$

Equation (2), which is used in the calculation of the phase χ , is the two-body amplitude for the free nucleon-nucleon interaction and needs to be modified for the nucleus-nucleus collisions. The present work looks into the modification of the free nucleon-nucleon amplitude in the medium. Total

absorption cross section for a wide range of proton-nucleus collisions is calculated using equation (6) and the nucleon-nucleon (NN) amplitude in equation (2) is modified until good agreement is found with the experiment. The modified nucleon-nucleon amplitude can be written as follows:

$$f_{NN,m} = f_m f_{NN} \quad (7)$$

where $f_{NN,m}$ is the nucleon-nucleon amplitude in the medium (nucleus), f_{NN} is the free NN cross section, and f_m represents the system and energy dependent functions defined in equation (8).

Thus, the renormalized amplitude is extracted directly from the experiment and is a reliable measure of the medium modifications. The medium multipliers defined in equation (7) for ion kinetic energy in a laboratory system (tlab) in A MeV, are given by

$$f_m = 0.1 \exp\left(\frac{-\text{tlab}}{12}\right) + \left[1 - \left(\frac{\text{dens}}{0.14}\right)^{1/3} \exp\left(-\frac{\text{tlab}}{\text{tden}}\right)\right] \quad (8)$$

where, for $A_T < 56$,

$$\text{tden} = 46.72 + 2.21 A_T - 2.25 \times 10^{-2} A_T^2 \quad (9)$$

and for $A_T \geq 56$,

$$\text{tden} = 100 \quad (10)$$

In equation (8), dens refers to the average density of the colliding system of projectile and target, with mass numbers A_P and A_T respectively, and is given by

$$\text{dens} = \frac{1}{2}(\rho_{A_P} + \rho_{A_T}) \quad (11)$$

where the density of a nucleus A_i ($i = P, T$) is calculated in the hard sphere model and is given by

$$\rho_{A_i} = \frac{A_i}{\frac{4\pi}{3} r_i^3} \quad (12)$$

where the radius of the nucleus r_i is defined by

$$r_i = 1.29 (r_i)_{rms} \quad (13)$$

The root-mean-square radius, $(r_i)_{rms}$, is obtained directly from the experiment (ref. 10) after “subtraction” of the nucleon charge form factor (ref. 4).

Equation (6) was also modified to account for the Coulomb force in the proton-nucleus absorption cross sections. The Coulomb force has significant effects at low energies, is less important as the energy increases, and practically disappears for energies above 50 A MeV.

For nucleus-nucleus collisions, the Coulomb barrier (B_c) energy is given by

$$B_c = \frac{1.44 Z_P Z_T}{R} \quad (14)$$

with

$$R = r_P + r_T + \frac{1.2(A_P^{1/3} + A_T^{1/3})}{E_{cm}^{1/3}} \quad (15)$$

where Z_P and Z_T are charge numbers for the projectile and target, respectively, and R is the radial distance between their centers, E_{cm} is the colliding system center of mass energy in A MeV. These expressions also are kept for the proton-nucleus collisions to have a unified picture of any colliding system (ref. 11). However, equation (15) overestimates the radial distance between proton-nucleus collisions, and hence equation (14) underestimates the Coulomb energy between them. To compensate for this, in such cases it is necessary to multiply equation (14) by the following factor, which is then the Coulomb multiplier to equation (6):

$$X_{\text{Coul}} = (1 + C_1/E_{cm}) (1 - C_2 B_c/E_{cm}) \quad (16)$$

For $A_T < 56$,

$$\left. \begin{aligned} C_1 &= 6.81 - 0.17A_T + 1.88 \times 10^{-3} A_T^2 \\ C_2 &= 6.57 - 0.30A_T + 3.6 \times 10^{-3} A_T^2 \end{aligned} \right\} \quad (17)$$

and for $A_T \geq 56$,

$$\left. \begin{aligned} C_1 &= 3.0 \\ C_2 &= 0.8 \end{aligned} \right\} \quad (18)$$

The above formalisms are also used for nucleus-nucleus collisions. Care is necessary in accounting for the surface and Coulomb effects of the projectile. The surface effects are taken into account by the parameter t_{den} in equation (8). For alpha particle and carbon projectiles, where surface is prominent, the values of 300 and 100, respectively, are used for t_{den} . For rest of the nuclei considered, where there is not much variation in the surface, a t_{den} value of 200 has been used.

The Coulomb energy for nucleus-nucleus collisions is also taken into account by equation (16). The parameter $C_1 = 0$ for all nucleus-nucleus collisions. The best values of the parameter C_2 used in the integration of equation (6) for various systems considered is as follows:

$$\begin{aligned} C_2 &= 0.625 \text{ for } \alpha + C \\ &= 0.3 \text{ for } \alpha + Pb \\ &= 0.35 \text{ for } C + C \\ &= 0.5 \text{ for all other systems} \end{aligned}$$

3. Discussion and Results

Figures 1–4 show the results of our calculations of reaction cross sections for proton on carbon, aluminum, iron, and lead targets, respectively. The experimental data were taken from the compilation of references 12–21. The formalism presented here agrees very well with the experiment for all systems for the entire energy range. The medium amplitude multiplier equation (8) is shown in figure 5 and shows that as the energy increases the multiplier tends to a value of unity, as expected, since the Pauli effects responsible for the modification of the amplitude in

the medium become less important. It is also noted that as the target becomes heavier, the multiplier becomes smaller, indicating that here the Pauli effect plays a bigger role and gives greater reduction of the cross section in the medium as expected.

Figures 6–13 show the results for reaction cross sections for several nucleus-nucleus systems. The data for $\alpha + C$ system (fig. 6) was obtained from references 13, 14, 19, and 20. Extensive data available for $C + C$ system (fig. 8) were taken from references 13, 14, 20, and 21. For the remainder of the figures, data were collected from the compilation of data sets from references 13–17 and 21. The agreement with the experiments is very good. It is satisfying to know that the in-medium, nucleon-nucleon amplitudes derived here give good results for the nucleus-nucleus collisions as well.

The medium multiplier, equation (8), is simple to use and gives reliable results for the nucleon-nucleon amplitude in the medium down to very low energies.

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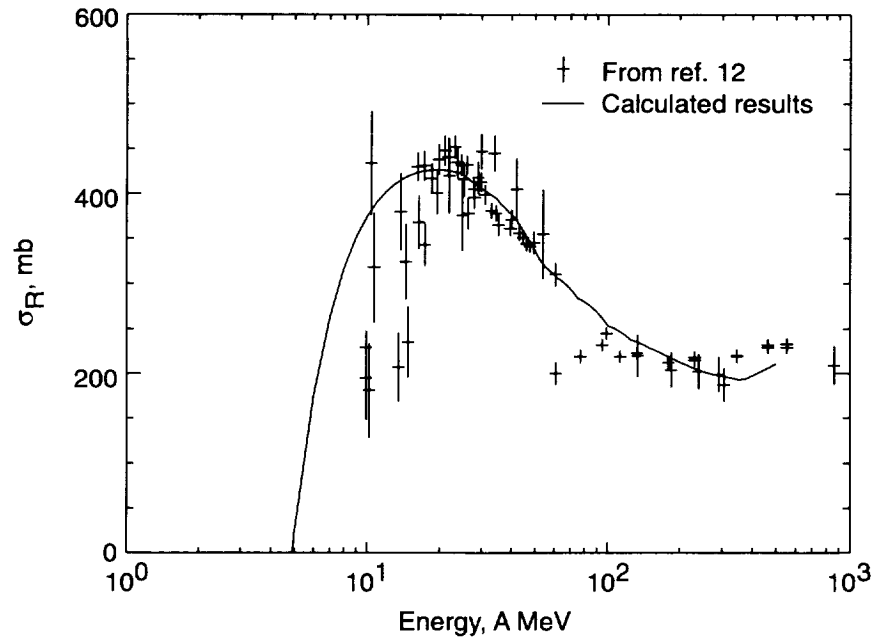


Figure 1. Absorption cross section for $p + {}^{12}_6\text{C}$ collision as a function of energy.

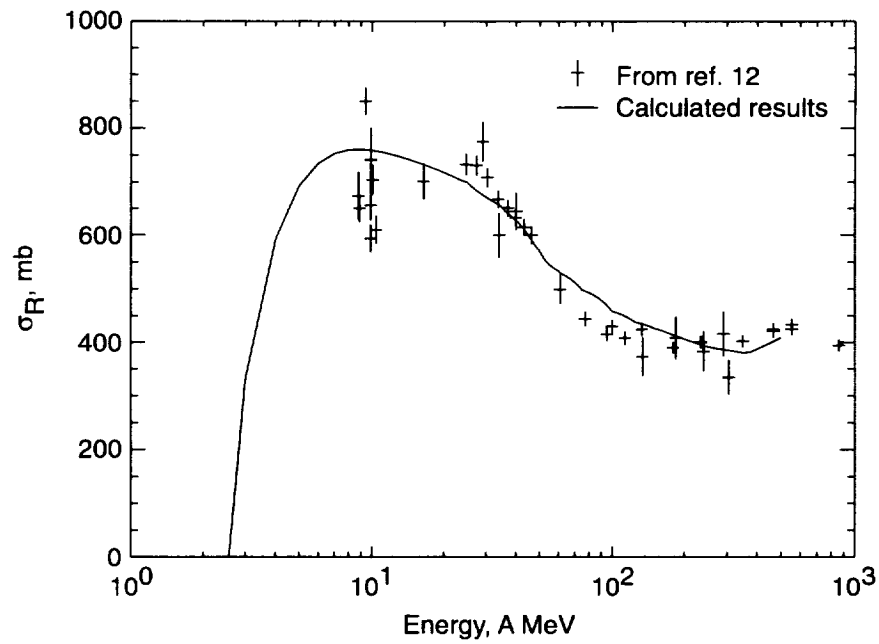


Figure 2. Absorption cross section for $p + {}^{27}_{13}\text{Al}$ collision as a function of energy.

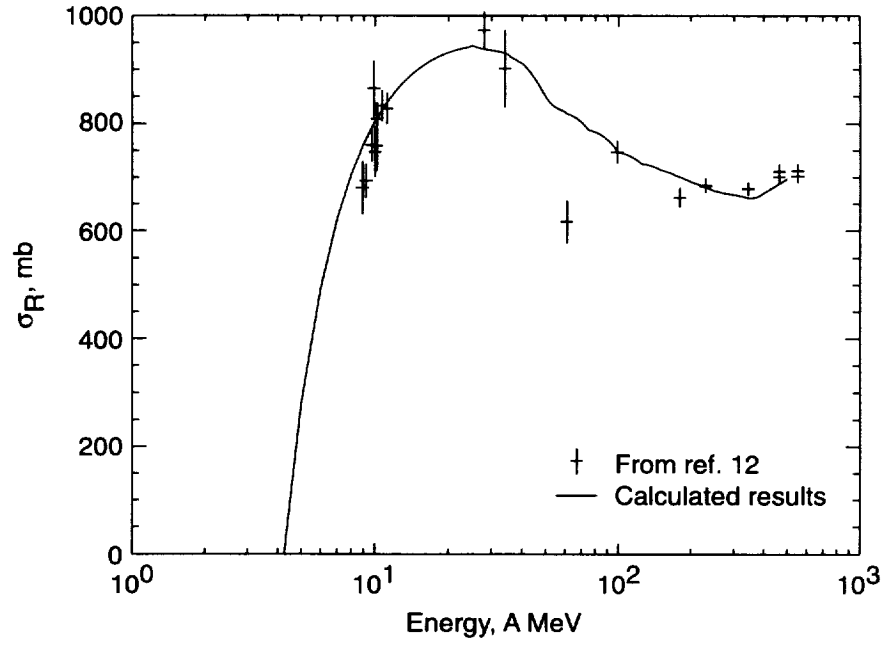


Figure 3. Absorption cross section for $p + {}^{\text{nat}}\text{Fe}$ collision as a function of energy.

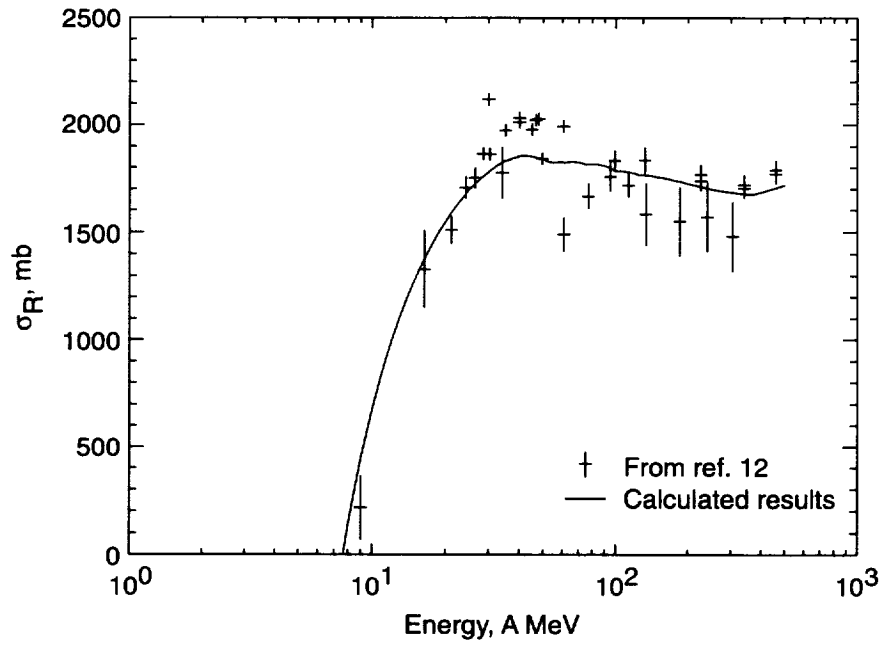


Figure 4. Absorption cross section for $p + {}^{208}_{82}\text{Pb}$ collision as a function of energy.

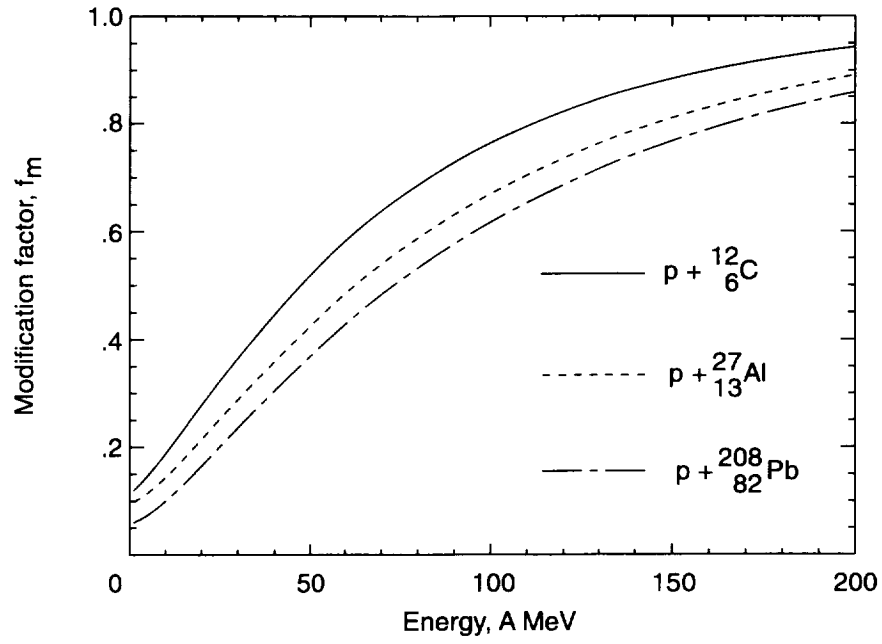


Figure 5. Medium multiplier factors (f_m), equation (8), as a function of energy for various systems.

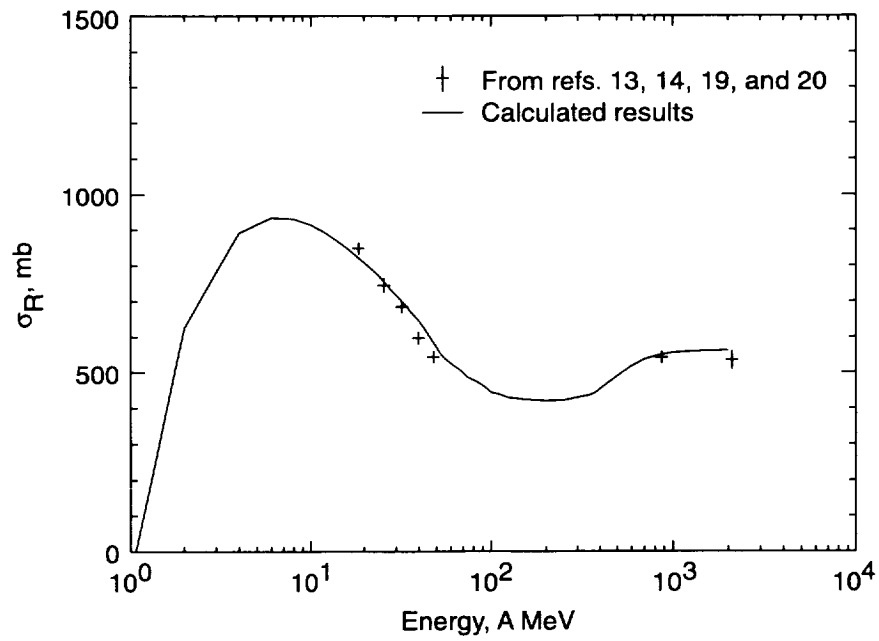


Figure 6. Absorption cross section for $\alpha + {}^{12}_6\text{C}$ collision as a function of energy.

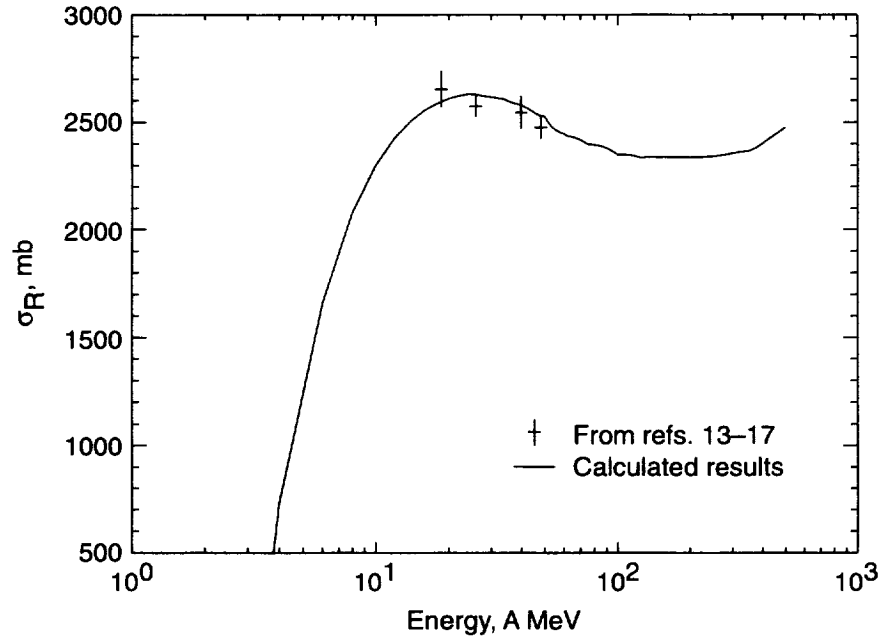


Figure 7. Absorption cross section for $\alpha + {}^{208}_{82}\text{Pb}$ collision as a function of energy.

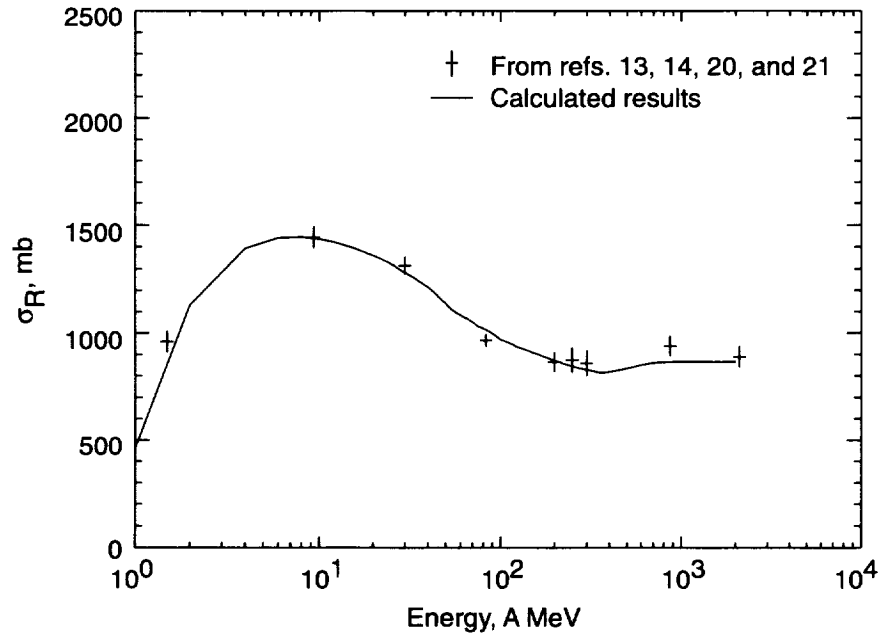


Figure 8. Absorption cross section for ${}^{12}_6\text{C} + {}^{12}_6\text{C}$ collision as a function of energy.

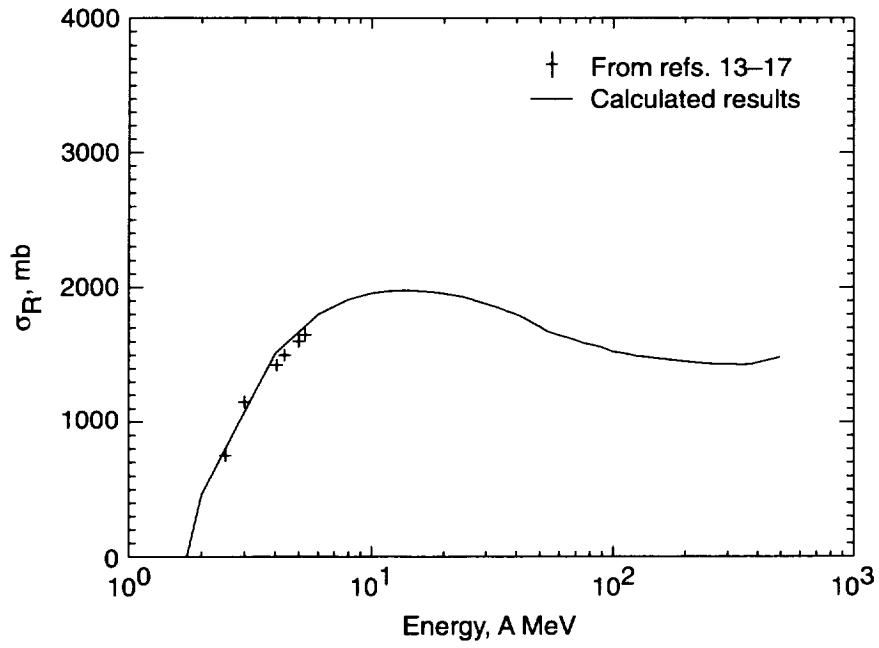


Figure 9. Absorption cross section for $^{16}_8\text{O} + ^{28}_{14}\text{Si}$ collision as a function of energy.

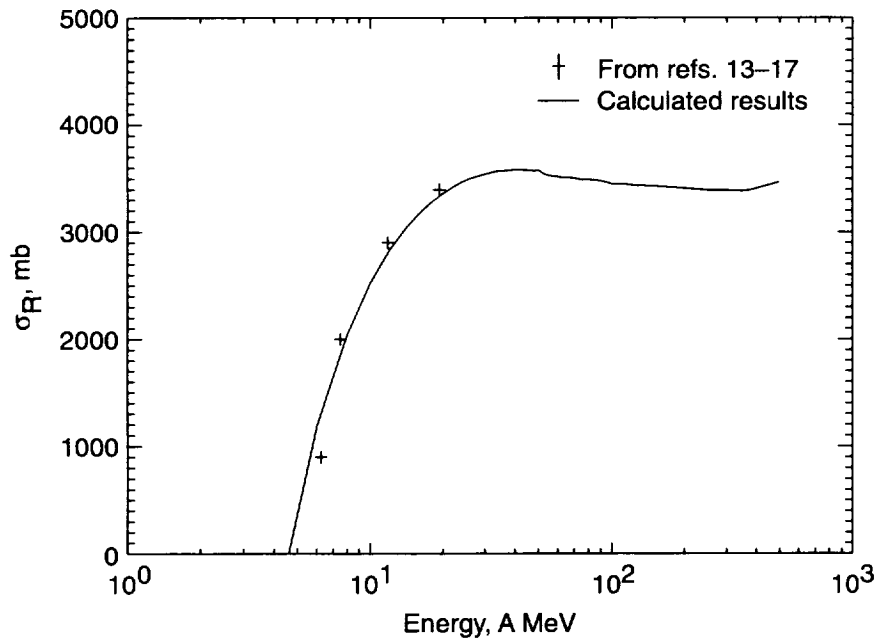


Figure 10. Absorption cross section for $^{16}_8\text{O} + ^{208}_{82}\text{Pb}$ collision as a function of energy.

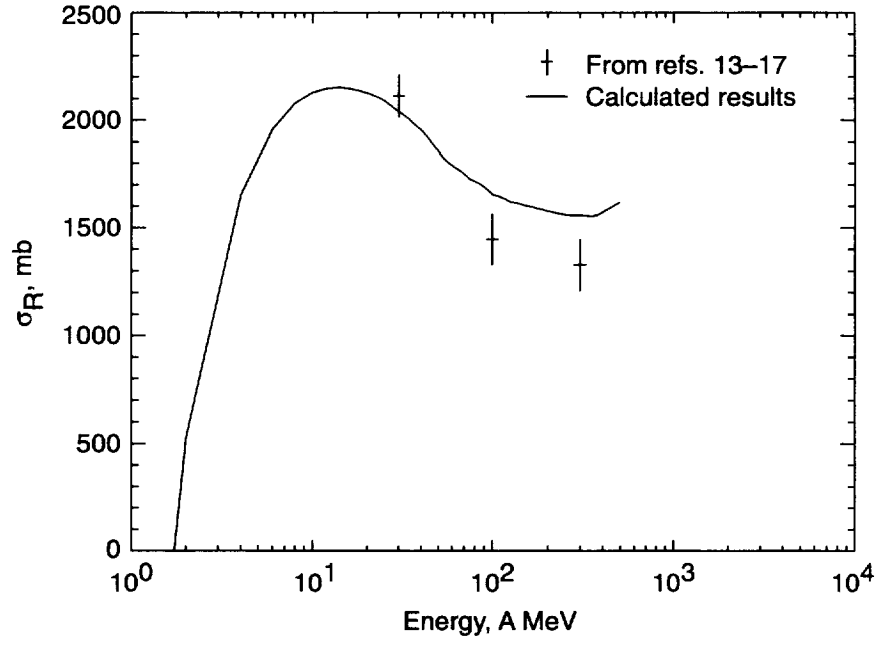


Figure 11. Absorption cross section for $^{20}_{10}\text{Ne} + ^{27}_{13}\text{Al}$ collision as a function of energy.

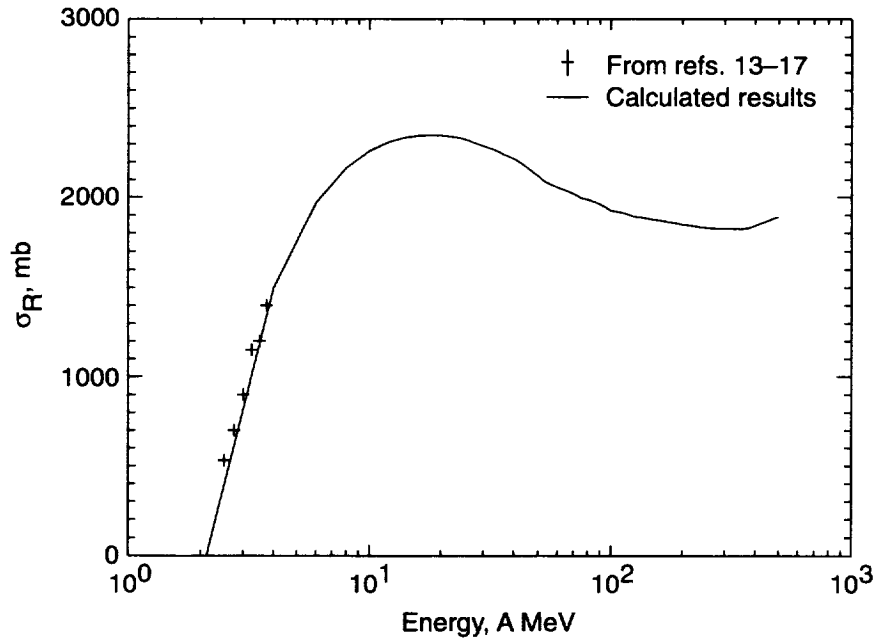


Figure 12. Absorption cross section for $^{20}_{10}\text{Ne} + ^{40}_{20}\text{Ca}$ collision as a function of energy.

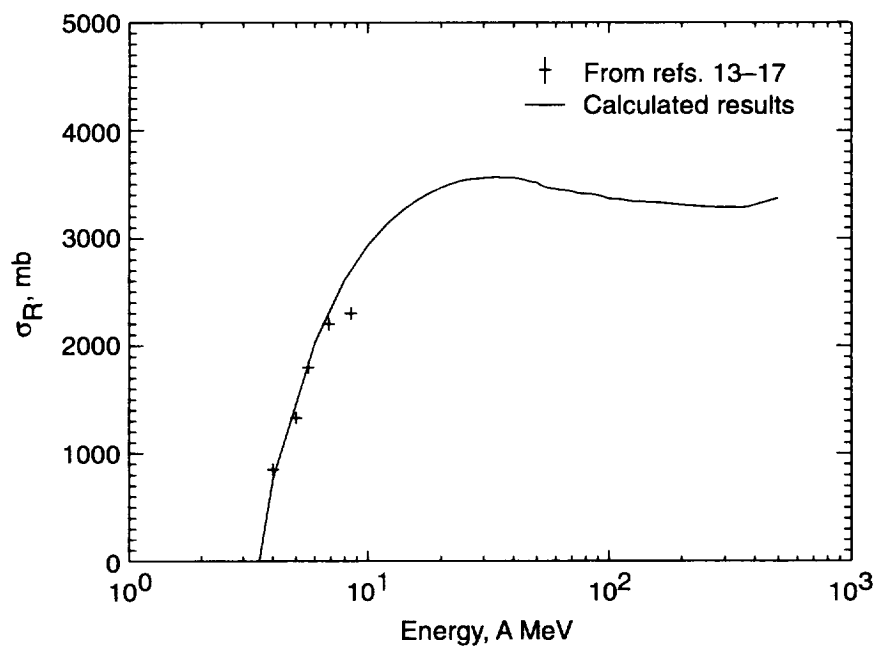


Figure 13. Absorption cross section for $^{40}_{18}\text{Ar} + ^{107}_{47}\text{Ag}$ collision as a function of energy.

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